A TRIBUTE TO THE CONTRIBUTION OF GÉRARD LALLEMENT TO STRUCTURAL DYNAMICS

Scott Cogan[1]
Université de Franche-Comté, France

François M. Hemez[2]
Los Alamos National Laboratory, U.S.A.

Michael Link[3]
University of Kassel, Germany

John E. Mottershead[4]
University of Liverpool, U.K.

Ben Wada[5]
Jet Propulsion Laboratory, U.S.A.

Charles Zhang[6]
Renault, S.A., France

ABSTRACT

The Society for Experimental Mechanics and the International Modal Analysis Conference recognize the remarkable contribution to experimental mechanics, mechanical engineering and structural dynamics of Professor Gérard Lallement, from the University of Franche-Comté, France. A special session is organized during the IMAC-XX to outline the many achievements of Gérard Lallement in the fields of modal analysis, structural system identification, the theory and practice of structural modification, component mode synthesis and finite element model updating. The purpose of this publication is not to provide an exhaustive account of Gérard Lallement's contribution to structural dynamics. Numerous references are provided that should help the interested reader learn more about the many aspects of his work. Instead, the significance of this work is illustrated by discussing the role of structural dynamics in industrial applications and its future challenges. The technical aspects of Gérard Lallement's work are illustrated with a discussion of structural modification, modeling error localization and model updating.

1. INTRODUCTION

The Society for experimental mechanics has decided to recognize the contribution to experimental mechanics, mechanical engineering and structural dynamics of exceptionally talented individuals. To do so, SEM will attempt to organize special honorary sessions during its annual International Modal Analysis Conference (IMAC).

Beyond the recognition by the community of a particularly remarkable achievement or a life-long contribution, this initiative is also aimed at teaching new generations of structural dynamicists of past achievements that they may not be aware of as well as past mistakes and lessons learned.

The first such honorary session coincides with the 20th anniversary of the IMAC, held in Los Angeles, California, in February 2002. It is dedicated to the work of Professor Gérard Lallement. In September 2001, Gérard Lallement retired from his positions of Professor and Director of the Laboratory of Applied Mechanics (LMARC) that he contributed to establish more than 40 years ago, at the University of Franche-Comté, France.

2. BIBLIOGRAPHICAL BACKGROUND

Gérard Lallement started a 40-year long career after earning a Ph.D. in mechanical engineering from the University of Franche-Comté in 1961. His research interests have included the study of quartz crystals for eight years; the study of magnetic-mechanical coupling of various metals and alloys for another eight years; and, finally, what is currently known as structural dynamics for the past 25 years. The results of his research for the period 1961-2001 have been published in 62 journal publications, 126 conference communications and countless internal, European, NATO, industrial and contract reports.

---

[1] CNRS senior research fellow, Laboratory of Applied Mechanics, **scott.cogan@univ-fcomte.fr**
[2] Technical staff member, Engineering Sciences and Applications Division, ESA-WR, **hemez@lanl.gov**
[3] Professor, Lightweight Structures and Structural Mechanics Laboratory, **link@hrz.uni-kassel.de**
[4] Professor, Department of Engineering, Mechanical Engineering Division, **j.e.mottershead@liverpool.ac.uk**
[5] Consultant/retired, Jet Propulsion Laboratory, **ben_wada@earthlink.net**
[6] Senior technical staff member, Renault Technology Center, **charles.zhang@renault.com**

In this section, four aspects of Gérard Lallement's career are briefly summarized. First, his research interests and teaching experience are discussed. The development of the LMARC is mentioned next. Finally, the international aspects of his career are discussed.

2.1 Research Interests

The expertise developed by Gérard Lallement's diverse research interests is vast and multidisciplinary. In the following, we attempt to provide the reader with an illustration of some of the problems that he has extensively studied throughout the period 1961-2001: **(1)** Conception of high-precision machine tools for watch manufacturing; **(2)** Study of thermo-elastic abnormalities in Fe-Ni alloys; **(3)** Study of the inelastic behavior and magnetic-mechanical coupling of metals; **(4)** Homogenization techniques for plate structures with discontinuities; **(5)** Inverse problems in mechanical engineering (modal identification, model updating, structural optimization); **(6)** Development of modal testing techniques; **(7)** Study of multiple aspects of rotor dynamics; **(8)** Development of techniques for structural modification such as re-analysis techniques, spectral decomposition, pseudo-testing and fictitious boundary conditions; **(9)** Modeling of structures in the medium frequency range and acoustics; **(10)** Sensitivity and selective sensitivity study; and **(11)** Component mode synthesis, reduction basis and model condensation for linear and nonlinear structures.

A distinct characteristic of these research activities is the early—and, in many ways, visionary—understanding that the analysis of experimental data sets and numerical models must be assisted with the development and implementation of software tools. This has lead to several generations of software packages for structural modal identification such as MODAN and an outstanding MATLAB™-based package for structural modeling and optimization currently known under the acronym of AESOP (Analytic and Experimental Structural Optimization Platform).

2.2 Teaching Experience

The second aspect of Gérard Lallement's career resides in his teaching activity. He has developed more than 36 years of experience teaching undergraduate and graduate courses that cover all aspects of the theory of elasticity, mechanical engineering and structural dynamics.

Throughout his teaching career, Gérard Lallement averaged an impressive 190-to-250 hours per year in the classroom and wrote several textbooks to support his courses. More specifically, the courses taught included solid mechanics; strength of materials; finite element modeling and analysis; mechanisms; structural dynamics; structural optimization; modal testing and identification; and acoustics.

Gérard Lallement always likes to mention that teaching and, conversely, learning from the students provided the memories of which he is the proudest. A quotation by Albert Einstein most certainly applies to Gérard Lallement's philosophy of teaching:

> *"It is the supreme art of the teacher to awaken joy in creative expression and knowledge."*

2.3 Development of the LMARC

In 1961, Professor René Chaléat and Gérard Lallement founded the Laboratory for Applied Mechanics (LMARC) in Besançon, France. At the time, it was the first such laboratory in France and among the first ones in Europe to be dedicated to the study of applied mechanics problems that brought structural dynamics at the center of its activities.

In many ways it was a risky endeavor to promote the development of a laboratory whose research activities were centered at the engineering and experimental aspects of what is now known as structural dynamics. The reason is because applied mathematicians dominate the mechanics and acoustics communities in France as opposed to experimentalists. Nevertheless, the LMARC rapidly became a leader in France and Europe, actively participating in the development and diffusion of vibration testing and modeling techniques throughout the industry and educating several generations of structural dynamicists.

The success of the LMARC is, to a great extent, due to an aggressive pursuit of development through industrial collaboration and numerous contracts, a strategy engineered and carried out by Gérard Lallement. This strategy was, again, visionary because, back in the early 1960's, educational and research institutions were largely funded by the government and not so much by the industry. In 1966 and only four years after its creation, the LMARC was affiliated to the French National Center for Scientific Research (CNRS), which is the most prestigious recognition in France for a public research institution.

Other important factors that still contribute today to the laboratory's success reside in the world-class technical proficiency of its staff and the atmosphere of

friendship and open collaboration that reflect Gérard Lallement's personality.

Gérard Lallement headed the structural dynamics research team of the LMARC for 22 years, from 1975 to 1996. He was the LMARC's deputy director for 12 years and its director from 1995 to 2000. In 1993, the structural dynamics team headed by Gérard Lallement was the LMARC's most important team and it was composed of 10 staff members and 13 Ph.D. students. Today, the LMARC counts 47 staff members and more than 50 graduate and Ph.D. students. It is organized in five teams: **(1)** Structural dynamics; **(2)** Material behavior; **(3)** Material process modeling; **(4)** Micro-machines; and **(5)** Robotics. For completeness, some of the projects currently investigated by each research team are listed below. The list illustrates the diversity and multi-disciplinary aspects of the research performed at the LMARC:

(1) Structural Dynamics:

Robust design of structures; Robust component mode synthesis; Equivalent finite element modeling; MEMS and distributed active control; Optimal test planning; Identification and uncertainty of modal parameters; Wavelet modal analysis; and Engineering software development.

(2) Material Behavior:

Micro-structure and behavior of aluminum alloys; Modeling of crack propagation; Homogenization of composite materials; Thermo-magnetic-mechanic modeling of shape memory alloys; Control of shape memory alloy actuators; Modeling of internal stresses during MEMS manufacturing; Behavior of piezo-elastic and piezo-composite materials; and Experimental investigation of the mechanical properties of human skin.

(3) Material Process Modeling:

Optimization and process control of thin sheet forming; Prediction of forming defaults; Scaling effects during thin sheet forming; Numerical simulation of precision cutting; Multiple scale simulation of the densification of metal powders; and Parallel computing and transient analysis of large-scale processes.

(4) Micro-machines:

Process for the manufacturing of silicon films; Micro-actuators with contact interaction; Micro-

engines; Distributed micro-actuation and macro-scale integration; In-situ characterization of poly-crystal silicon; and Networks of acoustic micro-transducers.

(5) Robotics:

Micro-robots; Assembly of wooden structures; and Fuzzy logics applied to the analysis of measurements.

During his tenure as director of the LMARC, Gérard Lallement has promoted novel research and development directions, such as biomechanics, and actively contributed to the creation of the new micro-machines team. Through this evolution, his concern has always been to extend the LMARC's activity to reflect the ever-changing industrial needs, programs and environments while capitalizing on the laboratory's core competency in structural dynamics. The LMARC remains a flourishing institution and one of the premier centers for structural dynamics research in Europe.

2.4 International Recognition

Today, the contribution to structural dynamics and technical expertise of Gérard Lallement are recognized internationally. He has been an early supporter of national and international modal analysis meetings and contributed to organize several such meetings himself.

In addition, he has provided the motivation for the EMAUG (European Modal Analysis User Group) that contributed to the dissemination of technical knowledge in the fields of vibration testing and analysis for over 10 years. Gérard Lallement has attended the very first IMAC meetings and there has not been a year when students or researchers from the LMARC have not attended the IMAC since its creation in 1983. Similarly, he has been a supporter of the University of Leuven's International Seminar on Modal Analysis (ISMA) since its creation in 1986.

The research activities of Gérard Lallement have attracted interest internationally and he has established collaborations with colleagues from countries among which we cite China, India, Russia, Armenia, Poland, Romania, Hungary, The Czech Republic, Germany, Switzerland, Belgium, Tunisia, Morocco, The United Kingdom, The United States, Japan and South Korea.

3. INDUSTRIAL APPLICATIONS

In many ways, the research of Professor Gérard Lallement has contributed to pioneer techniques that are widely used in the industry today. This section illustrates some of the problems that he has studied and contributed to solve.

The techniques that are fields of active research nowadays and that have been pioneered by Gérard Lallement, among others, include experimental techniques for vibration testing, sensitivity study, re-analysis techniques and finite element model updating. In addition, Gérard Lallement has had the opportunity to work on countless industrial contracts and develop solutions to practical engineering problems. Some of the techniques proposed for improving the fidelity of finite element models are still widely used in the aerospace and automotive industries.

3.1 The Industrial Context

One such problem to which Gérard Lallement has devoted considerable attention over the past 30 years is the reduction of booming noise for automobiles. The booming noise originates from low frequency vibrations at, typically, less than 200 Hertz. Sources include vibration generated by the engine and external aerodynamic excitation of the automobile's structure. Although such vibration occurs at a frequency bandwidth outside the audible domain, it may happen that it brings to a resonance the volume of air trapped inside the car, then, inducing a high level of discomfort for the driver and passengers.

In order to reduce the time-to-market and the high costs of development, the trend in the automotive industry for the past two decades has been to reduce the number of real prototypes. It eliminates expensive testing cycles but the information that used to be obtained through physical testing must now be obtained through predictive modeling and analysis. A requirement for proving that numerical simulations can be useful during the preliminary design cycles is to improve and guarantee the accuracy of predictions. However, this generally turns out to be a difficult problem given the fact that structural and thermal models for automotive components—and even more so for entire systems—are very complex.
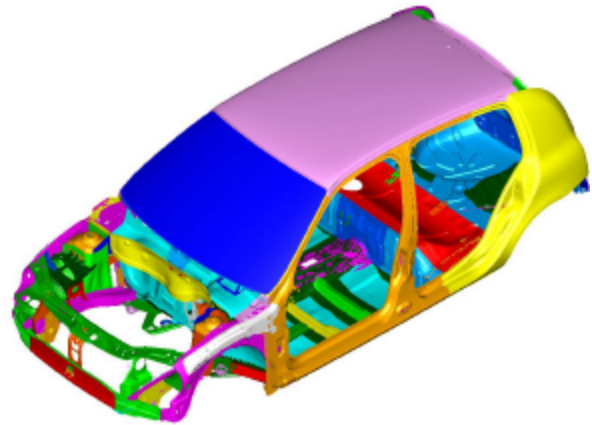
Figure 1. Automotive body-in-white.
The figure illustrates a conceptual model of Renault CLIO vehicle, courtesy of Renault, S.A. Such models are converted into computational meshes for thermal, structural, modal and acoustical simulations.

The most important trade-off to consider when developing techniques for improving the predictive accuracy of numerical simulations is the trade-off between accuracy and flexibility. Large-size numerical models are generally developed to ensure mesh convergence and make sure that all the dynamics of interest are captured. But larger numerical models become increasingly difficult to manipulate and analyze. On the other hand, modeling and analysis techniques must be flexible enough to allow changes and structural modifications, often performed during the early stages of a design cycle, to be rapidly taken into account. A compromise must therefore be sought between predictability and flexibility of the modeling and analysis practices. Gérard Lallement has contributed to provide a solution to this difficult problem through the development of numerical techniques for finite element model updating; sensitivity analysis; sub-structuring and component mode synthesis; model condensation; and re-analysis. Such tools are typically employed to develop other forms of models—such as conceptual and hybrid models—that complement the large physics-based models while answering the need for accurate, yet flexible, analysis procedures. The concepts of conceptual modeling and hybrid modeling are briefly discussed in the remainder.

3.2 Conceptual Modeling

Conceptual models are developed during the early design cycles to enable decision-making regarding a particular design when all the final details are not yet known. One typical question that requires the analysis of conceptual models is to identify which input parameters influence a performance criterion.

4

One technique for their development in the automotive industry that Gérard Lallement pioneered thirty years ago is model condensation. Typically, a large-size numerical model is condensed down to a subset of physical degrees of freedom while maintaining a direct connection to the input parameters of the design. Condensed models do not exhibit the same connectivity pattern as the original models—meaning that the physical load path is lost— but they theoretically capture the same dynamics and can be evaluated and modified very rapidly thanks to their reduced dimensionality. Among many other applications throughout his career, Gérard Lallement has successfully developed conceptual models for studying the design and performance of exhaust lines at Renault, S.A., France.

The main drawback of conceptual models—and condensed modeling in general—is that the physical meaning of internal components is lost. For example, the connection between several structural members of a truss assembly is generally represented as a point-wise connection between bending and shear elements. The structural mass, stiffness and energy dissipation characteristics of the actual connecting joint are subsequently lost by the process of approximating a complex connection with a simple mathematical representation. Of course, this tends to deteriorate the predictive accuracy. Errors of this type can be overcome by calibrating parameters of the reduced model. "Equivalent" parameters or material properties are sought in an effort to produce a better model—meaning, a reduced model that accurately predicts the global response of the system.

Generally, the calibration of numerical models is achieved through finite element model updating. Model updating consists of adjusting matrices or parameters of the numerical model according to an optimality criterion and optimization constraints. In the early 1970's, Gérard Lallement has been one of the first researchers to understand that numerical models could not be accurately reduced nor calibrated without, first, developing the updating technology. To emphasize this important contribution, the framework developed by Gérard Lallement and his colleagues for finite element model updating is outlined in section 4.

Today, model updating remains an important tool for simulation-based design and certification. More specifically, it is used extensively in the automobile and aerospace industries. New applications have even been developed in the 1980's and 1990's, such as structural health monitoring and damage detection. Over the years, the sensitivity-based techniques developed by Gérard Lallement have been refined by others and included in several commercial finite element modeling and analysis packages. Beyond calibration, model updating can also be used to identify sensitive design parameters; validate the rules implemented for numerical modeling; and develop databases in the early design cycles.

3.3 Hybrid Modeling

Hybrid models consist of combining numerical models with experimental measurements. For example, it might be convenient to develop a numerical model of a particular component or sub-system under design. However, modeling in detail the interaction of this component with its environment and the rest of the system might be too complicated. In addition, the physics-based modeling option might not offer the high flexibility previously discussed and required when changes to the conception are likely to occur during the early design cycles.

In this situation, it is advantageous to replace the coupling of the component with the rest of the system by experimental measurements. For example, transfer functions between the main system and the component can be experimentally estimated to any desired level of accuracy and used to represent the coupling. Component mode synthesis techniques have been used as the tool to enable the development of hybrid models. Industrial applications in the automotive and aerospace industries have demonstrated that hybrid models can be used for rapid prototyping while still providing highly accurate solutions.

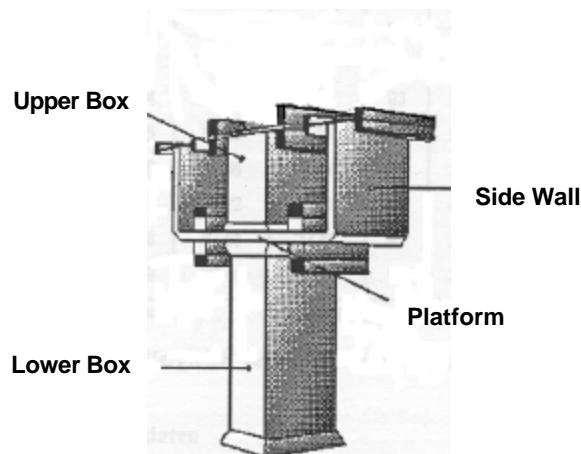## 4. ERROR LOCALIZATION AND UPDATING

After having discussed the significance of the work of Professor Gérard Lallement in the industrial context, some of the technical aspects of his work are now described. We illustrate them with the themes of modeling error localization and updating. The reason is because of the importance of this work in regards of today's research interests and industrial applications. The discussion that follows also illustrates the extent to which the updating framework developed by Gérard Lallement is general and provides solutions to many well-known issues such as the localization of modeling errors, the mitigation of ill-conditioning effects and sensitivity study. Some publications pertaining to this work are provided in References [B1] to [B7].

4.1 Model Updating and Related Issues

Most often, the dynamic response predicted by computational models is different from experimental

LA-UR-01-5820, Unclassified

data collected by instrumenting and monitoring the physical system. Common causes are: (1) Erroneous mathematical and modeling assumptions of the physical behavior; (2) Numerical discretization errors; (3) Erroneous assumptions of the type and location of the model parameters; (4) Erroneous assumptions for the test-analysis differences to be minimised; and (5) Unavoidable measurement errors. Other well-known difficulties of solving inverse test-analysis correlation problems include non-unique and non-physical solutions delivered by optimisation solvers.

The discussion outlines the formulation of model updating developed by Gérard Lallement and his colleagues. The discussion also addresses the mitigation of adverse ill-conditioning effects and the optimal selection of the types and locations of model parameters to be updated. These are important issues for the following reasons. First, ill-conditioning rapidly deteriorates the efficiency of numerical solvers and yields non-physical solutions. Techniques are available to reduce ill-conditioning at its source, such as adimensional analysis, but the application of such techniques to large computational models is somewhat limited because of practical and implementation considerations. Ill-conditioning can also be reduced by appropriately selecting which parameters of the model are adjusted and solving the inverse problem in a carefully selected subspace instead of the original space. This approach—originally proposed by Gérard Lallement—is illustrated next.

Figure 2. RESMOD laboratory test structure.
The figure illustrates a laboratory test structure instrumented to study the performance of modeling error localization and model updating techniques.
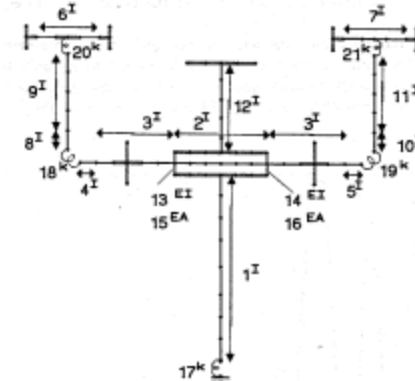
Figure 3. Modeling of the RESMOD structure.
The figure illustrates a finite element representation of the RESMOD structure. The system is discretized into spring and beam elements.

The second problem discussed here addresses the identification of modeling errors. Obviously, the parameters selected for updating should capture the nature and location of the modeling error. In the case of structural health monitoring, for example, the selection of parameters to update should reflect the type and location of the damage. This is difficult because both modeling error location and nature are unknown a priori. The localization and updating process is therefore an iterative procedure that could be qualified of "trial and error" if it were performed entirely at random. Generally, the localization is characterized by a qualitative selection of sub-domains—or by the definition of macro-elements—of a finite element model. The analyst's experience is often critical to identify which mechanisms or sub-domains carry the dominant error. This approach is however limited in cases where little experience is available or new designs are being assessed numerically. In contrast, Gérard Lallement has been amongst the first to develop semi-automatic procedures for identifying areas of the model that are responsible for the discrepancy between measured and predicted responses. Automatic model parameter selection procedures typically attempt to fulfil the following objectives: (1) Provide an initial evaluation of a parameter's feasibility to be used for model updating; (2) Reduce the number of effective parameters; and (3) Improve the conditioning of the estimation equations.

4.2 Model Updating Formulation

The parametric updating problem is generally described by a minimisation problem. The objective is to minimize a cost function $J(p)$ with respect to model parameters $\{p\}$

$$J(p) = \{\mathbf{e}_\mathbf{V}(\mathbf{p})\}^\mathrm{T} \{\mathbf{e}_\mathbf{V}(\mathbf{p})\} \qquad (4\text{-}1)$$

where the term **{e_V}** denotes a weighted residual vector that expresses a distance between a measurement vector **{v_M}** and a prediction vector **{v(p)}** such as

$$\{\mathbf{e}_\mathbf{V}(\mathbf{p})\} = [\mathbf{W}_\mathbf{V}](\{\mathbf{v}_\mathbf{M}\} - \{\mathbf{v(p)}\}) \qquad (4\text{-}2)$$

The measurement and prediction vectors typically collect features that represent the static or dynamic behavior of the structure. The most common choice is eigen-frequencies and mode shape vectors because experimental procedures are available to extract modal parameters from vibration measurements. Equations (4-1) and (4-2), nevertheless, do not restrict the formulation to modal data. Clearly, other quantities such as static deflection shapes, Ritz vectors or frequency response functions can be used to define the weighted residual vectors **{e_V}**.

Equation (4-2) indicates that the prediction vectors **{v(p)}** depend on the design parameters **{p}** because they collect features that characterize the response predicted by the finite element model. For example, if resonant frequencies are used, the feature vector is defined as

$$\{\mathbf{v(p)}\} = \begin{Bmatrix} s_1^2(p) \\ s_i^2(p) \\ \vdots \\ s_j^2(p) \end{Bmatrix} \qquad (4\text{-}3)$$

where $s_j(p)$ denotes the jth radial frequency extracted from the conservative finite element model

$$[\mathbf{K(p)}]\{\mathbf{u}_\mathbf{j}\} = s_j^2[\mathbf{M(p)}]\{\mathbf{u}_\mathbf{j}\} \qquad (4\text{-}4)$$

The vector **{p}** of design parameters collects the physical or non-physical parameters that are optimized in order to minimize the cost function *J(p)*. Clearly, the main question of parameter selection is to identify which combinations of these parameters are responsible for explaining the modeling error.

4.3 Linearization and Numerical Resolution

In general the model vector **{v(p)}** represents a nonlinear function of the parameters, resulting in a nonlinear minimization problem. One of the techniques to solve the nonlinear optimization is to expand the model vector of features into a Taylor series truncated after the linear term, as outlined below.

The solution procedure consists of writing the optimum design parameter vector **{p}** as a known contribution **{p_o}** and an unknown contribution **{dp}**. The known contribution represents the current design and the unknown contribution represents a deviation from this current state. The objective is to change the design from **{p_o}** to (**{p_o}+{dp}**) in such a way that the cost function *J(p)* is reduced as much as possible. To account for this decomposition, the residual vector **{e_V}** can be written as

$$\{\mathbf{e}_\mathbf{V}(\mathbf{p})\} = [\mathbf{W}_\mathbf{V}](\{\mathbf{v}_\mathbf{M}\} - \{\mathbf{v(p}_\mathbf{o} + \mathbf{dp)}\}) \qquad (4\text{-}5)$$

The first-order linearization of the residuals is written as

$$\{\mathbf{e}_\mathbf{V}(\mathbf{p})\} \approx [\mathbf{W}_\mathbf{V}](\{\mathbf{v}_\mathbf{M}\} - \{\mathbf{v}_\mathbf{o}\}) - [\mathbf{S}][\mathbf{dp}] \qquad (4\text{-}6)$$

where the vector of response features is evaluated at the current design point, namely, **{v_o}={v(p_o)}**, and the matrix of response feature sensitivity is defined as

$$[\mathbf{S}] = [\mathbf{W}_\mathbf{V}]\left[\frac{\mathbf{dv}}{\mathbf{dp}}(\mathbf{p}_\mathbf{o})\right] \qquad (4\text{-}7)$$

Finally, a minimum cost function is obtained locally by correcting the current design **{p_o}** with an increment **{dp}** obtained by solving the following system of linear equations

$$\{\mathbf{dp}\} = [\mathbf{S}_\mathbf{F}]^{-1}[\mathbf{S}]^\mathrm{T}[\mathbf{W}_\mathbf{V}](\{\mathbf{v}_\mathbf{M}\} - \{\mathbf{v}_\mathbf{o}\}) \qquad (4\text{-}8)$$

where the matrix **[S_F]** to be inverted is called the Fisher information matrix and defined as

$$[\mathbf{S}_\mathbf{F}] = [\mathbf{S}]^\mathrm{T}[\mathbf{S}] \qquad (4\text{-}9)$$

It is assumed in the derivation of equation (4-8) that the residual weighting matrix **[W_V]** is symmetric. In the applications documented in the work of Gérard Lallement, the residual weighting matrix is generally diagonal and its coefficients are defined as scaling factors between the components of the vector **{v(p)}**. Improvements to the original formulation summarized in equations (4-1) to (4-9) have since been proposed. For example, a penalty term can be added to the cost function *J(p)* in order to regularize the solution procedure and reduce the effects of numerical ill-conditioning. Another example is Bayesian parameter inference that follows the same basic formulation with the main difference being that the residual weighting

matrix is defined as the inverse of the residual's covariance matrix

$$\left(\left[\mathbf{W_V}\right]^T\left[\mathbf{W_V}\right]\right)^{-1} = \mathbf{E}\left[\left(\{\mathbf{v_M}\}-\{\mathbf{v_o}\}\right)\left(\{\mathbf{v_M}\}-\{\mathbf{v_o}\}\right)^T\right] \quad (4\text{-}10)$$

Clearly, the matrix **[S$_F$]** inverted in equation (4-8) depends on the type of residuals and optimisation parameters defined. The parameters can represent physical variables of the model, matrix entries or perturbation matrices. The latter case is the procedure proposed by Gérard Lallement. Typically, several sub-domains are defined geometrically and represented by partitioned sub-matrices. Then, the global finite element matrices are adjusted by multiplying each sub-matrix by a scaling parameter dp_k. Equation (4-11) illustrates this correction strategy where the stiffness matrix is decomposed into N_S sub-domains

$$\left[\mathbf{K}(\mathbf{p_o}+\mathbf{dp})\right] = \left[\mathbf{K}(\mathbf{p_o})\right] + \left[\mathbf{DK}(\mathbf{p_o})\right]$$
$$\left[\mathbf{DK}(\mathbf{p_o})\right] = \sum_{k=1...N_S} dp_k \left[\mathbf{K^{(k)}}(\mathbf{p_o})\right] \quad (4\text{-}11)$$

where **[K$^{(k)}$]** denotes the kth sub-domain's partitioned stiffness sub-matrix. It is emphasized that the weighted information matrix **[S$_F$]** is the mechanism through which numerical ill-conditioning is introduced. For one thing, it involves inner products of the columns of the original sensitivity matrix **[S]**, as can be observed from equation (4-9). Thus the condition number of matrix **[S$_F$]** is equal to the square of the condition number of matrix **[S]**, which greatly amplifies any ill-conditioning introduced by the original sensitivity matrix. The contribution of Gérard Lallement to procedures for optimal parameter selection is centered at the study of the conditioning and dimension of the sensitivity matrix, which plays an important role in the solution's accuracy / uniqueness.

4.4 Subspace Correction Approach

One of the most efficient methods of parameter selection and ill-conditioning mitigation was developed by Gérard Lallement. It is referred to as the Subspace Correction Method (SCM) in the following.

The principle of SCM is to extract from a large number of possible correction parameters denoted by N_P—the dimension N_P also identifies the number of columns of the weighted sensitivity matrix **[S]**—the smallest subset of parameters that best represents the test-analysis discrepancy. In order to illustrate the main steps of the parameter selection procedure, the correction step (4-8) is denoted as

$$\left[\mathbf{S_F}\right]\{\mathbf{dp}\}=\{\mathbf{r}\}$$
$$\{\mathbf{r}\}=\left[\mathbf{S}\right]^T\left[\mathbf{W_V}\right]\left(\{\mathbf{v_M}\}-\{\mathbf{v_o}\}\right) \quad (4\text{-}12)$$

The main idea of SCM is to implement a multiple-step localization procedure as outlined in the following. In the first localization step, equations (4-12) are solved using one column **{s$_k$}** of the Fisher information matrix at a time. This yields N_P least-squares solutions

$$dp_k^{(1)} = \left(\{\mathbf{s_k}\}^T\{\mathbf{r}\}\right)/\left(\{\mathbf{s_k}\}^T\{\mathbf{s_k}\}\right) \quad (4\text{-}13)$$

The quality of each solution is then assessed with an indicator of error defined as

$$e_k^{(1)} = 1 - \left(\left\|dp_k^{(1)}\{\mathbf{s_k}\}\right\|/\|\mathbf{r}\|\right) \quad (4\text{-}14)$$

where the symbol || || denotes the Euclidean norm—or L^2 norm—of a vector or matrix quantity. Such indicator measures the error associated to the resolution of equation (4-12) when it is resolved in the subspace defined by the kth column, as opposed to solving it in the original space defined by all columns of **[S$_F$]**. The smallest error identifies the column that is the most important one to solve the system of equations (4-12). This index is denoted by *L1* in the remainder. The contribution of the test-analysis discrepancy vector **{r}** is largest in the direction spanned by the $L1^{th}$ column **{s$_{L1}$}**. The $L1^{th}$ column and the corresponding parameter are therefore selected for correction.

Next, a second localization step is initiated. The $L1^{th}$ column previously identified is retained throughout this second step and subspace correction systems of dimension 2 are defined by repeating the previous procedure *(N$_P$-1)* times. Linear systems of the form

$$\{\mathbf{dp_k^{(2)}}\}=\left(\left[\mathbf{S_k^{(2)}}\right]^T\left[\mathbf{S_k^{(2)}}\right]\right)^{-1}\left[\mathbf{S_k^{(2)}}\right]^T\{\mathbf{r}\} \quad (4\text{-}15)$$
$$\left[\mathbf{S_k^{(2)}}\right]=\left[\{\mathbf{s_{L1}}\}\{\mathbf{s_k}\}\right]$$

are solved where the correction matrix of the second step and k^{th} iteration—denoted by **[S$_k$$^{(2)}$]**—contains the $L1^{th}$ column augmented with one of the *(N$_P$-1)* remaining columns. The *(N$_P$-1)* solution errors of the second localization step are calculated by

$$e_k^{(2)} = 1 - \left(\left\|\left[\mathbf{S_k^{(2)}}\right]\{\mathbf{dp_k^{(2)}}\}\right\|/\|\mathbf{r}\|\right) \quad (4\text{-}16)$$

The same selection procedure as previously outlined is carried out. The smallest error indicator identifies the subspace *(L1; L2)* in which the discrepancy vector **{r}** exhibits the largest contribution. It is therefore the best candidate subspace of dimension 2 for the resolution of the original system of equations (4-12). At
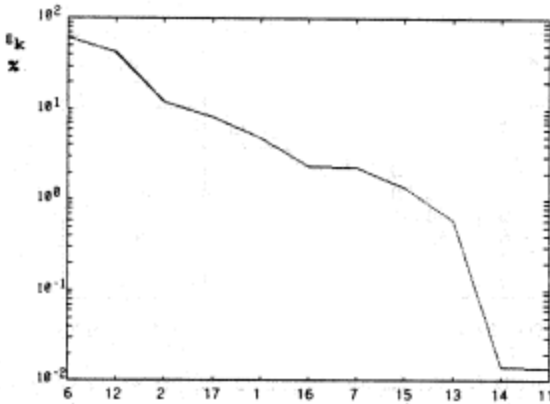
8

the end of the second localization step, the $L1^{th}$ and $L2^{th}$ design parameters are selected for correction and kept for the remainder of the analysis.

Obviously, the procedure can be repeated several times until the error indicators have decreased enough compared to the original values obtained at step 1. If p localization steps have been carried out, which leads to the selection of the $L1^{th}$, $L2^{th}$ … Lp^{th} input parameters, then, the $(p+1)^{th}$ error localization step is guided by the resolution of the following equations

$$\{\mathbf{dp}_k^{(p+1)}\} = \left(\left[\mathbf{S}_k^{(p+1)}\right]^T \left[\mathbf{S}_k^{(p+1)}\right]\right)^{-1} \left[\mathbf{S}_k^{(p+1)}\right]^T \{\mathbf{r}\}$$
$$\left[\mathbf{S}_k^{(p+1)}\right] = \left[\{\mathbf{s}_{L1}\}\{\mathbf{s}_{L2}\}\cdots\{\mathbf{s}_{Lp}\}\{\mathbf{s}_k\}\right] \qquad (4\text{-}17)$$
$$e_k^{(p+1)} = 1 - \left(\left\|\left[\mathbf{S}_k^{(p+1)}\right]\{\mathbf{dp}_k^{(p+1)}\}\right\|/\|\mathbf{r}\|\right)$$

where the index k takes the $(N_P\text{-}p)^{th}$ remaining values different from $L1$, $L2$ … Lp. As before, the smallest error indicator identifies which column should be added to the already available p columns to obtain the best subspace of dimension $(p+1)$ for the resolution of the system of equations (4-12).

Figure 4. RESMOD parameter error localization.
The figure illustrates the distribution of error indicator e_k as a function of the input parameter p_k. Because the error is shown on a logarithmic scale, correcting the first three parameters 6, 12 and 2 is sufficient to account for the quasi-totality of the modeling error.

The SCM selection of the model's parameters also provides a natural reduction of ill-conditioning. This is because ill-conditioning generally manifests itself through the combination of columns $\{\mathbf{s}_k\}$ that do not add any significant knowledge to solve the parametric correction system (4-12). Gérard Lallement and his co-workers have also investigated other strategies for optimal parameter selection. References [B2] and [B7], for example, detail other techniques that exploit the mode orthogonality and perturbation theory.

5. STRUCTURAL PSEUDO-TESTING

In this section, a second illustration of Professor Gérard Lallement's technical contribution to the field of structural dynamics is provided. Here, the themes of fictitious modifications and pseudo-tests are discussed. These are tools that can be used for increasing the knowledge space of a particular system / model without necessarily requiring additional instrumentation and measurement campaigns. Publications pertaining to this work are provided in References [C1] to [C6].

5.1 Fictitious Modifications and Pseudo-tests

The idea that new information, not available directly from a vibration test, might become accessible by processing the same measurements must be the product of lateral thought. In this part of the tribute to Gérard Lallement's research, we concentrate on a single equation, explain its significance in finite element model updating and draw attention to its potential in other areas of useful engineering application. Gérard Lallement's work with measured frequency responses to predict vibration behavior under different test configurations was done mainly to address problems of ill-conditioning and non-uniqueness. An important point is that a predictive model is produced.
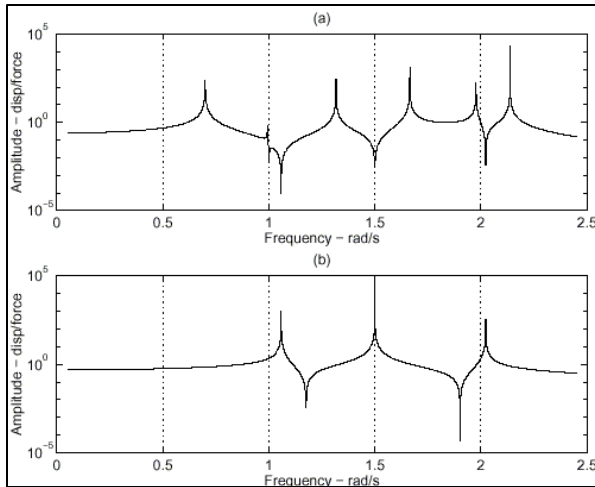
In principle, one might wish to carry out many vibration tests, under different conditions, so that a structure is excited in many linearly independent modes. In practice, however, the availability of industrial machinery and structures for vibration testing is usually extremely limited, often because of the cost of lost production while a machine is taken out of service for testing. Thus it is important both technically and commercially to maximize the usefulness of any measured data.

5.2 Performing Fictitious Modifications

One way to modify a structure would be to attach either a grounded spring or lumped mass at a point. A small modification would have very little effect on the dynamic behavior, though the effect would generally increase with the size of the mass and spring. Clearly the greatest effect is achieved when the magnitudes of the mass and stiffness approach infinity. In that case the structure would be grounded at the attachment point. In a numerical model, this is represented by the removal of the pth row and column of the stiffness and mass matrices, where the index p denotes the

9

attachment point. The mass and stiffness matrices obtained after deleting the pth row and column are denoted by **[K$_{pp}$]** and **[M$_{pp}$]**, respectively. The zeros of determinant $det([K_{pp}]-s^2[M_{pp}])$ define the antiresonances of the point receptance $h_{pp}(s)$ and interlace the poles (natural frequencies) of the system. We assume for reasons of simplicity that the system is undamped; the same procedure can of course be applied to a damped system but the poles and zeros in that case are generally complex.

In the case of a cross-receptance function $h_{pq}(s)$, the locations of antiresonance are given from the zeros of the equation $det([K_{pq}]-s^2[M_{pq}])=0$ where matrices **[M$_{pq}$]** and **[K$_{pq}$]** of the modified system are obtained by deleting the pth row and qth column of the original mass and stiffness matrices, respectively. The locations of antiresonance of the point-receptance and cross-receptance curves represent, in a sense, the extreme cases of point modification. The point-receptance antiresonances become the natural frequencies of the system grounded at the pth location. However, the locations of cross-receptance antiresonances cannot be obtained as natural frequencies by a passive modification. Nevertheless, all the antiresonances are potentially available as fictitious modifications.

Figure 5. Illustration of receptance data.
The top figure illustrates the cross-receptance function of a system. The bottom figure shows the receptance of a modified system obtained by grounding one of the degrees-of-freedom. The antiresonances of the original system become the poles of the modified system.

The first paper from the University of Besançon on structural modification appeared in 1989 and dealt with the problem of separating close modes in an almost square plate [C1]. The formula for selecting a spring to assign a particular natural frequency was given in terms of the natural frequencies of the original system and its eigenvectors at the modification coordinate. After separating the close modes, a finite element model of the plate was corrected by the sensitivity method previously outlined. At the 1992 IMAC, the first paper advocating the use of antiresonances in model updating appeared [C2]. As explained above the antiresonances can be thought of as a particular unit-rank modification. Gérard Lallement and Scott Cogan showed how the eigenvalues (antiresonances) of the system **([K$_{pq}$]; [M$_{pq}$])** and its eigenvectors could be determined from vibration measurements (see Reference [C2] for details). Furthermore, they introduced the new concept of the dynamic flexibility, or receptance matrix **[H$_{pq}$]**, having poles at the antiresonance locations of $h_{pq}(s)$.

5.3 Performing Pseudo-tests

Starting from the receptance equation of the original system, a constraint $x_p=0$ can be applied at the pth degree-of-freedom and across all frequencies by the application of a control force $f_q=-(H_{pj}/H_{pq})f_j$. This control force f_q may then be eliminated to obtain the following system of equations that establishes the relationship between the input forces f_j for all degrees-of-freedom j, except at the qth location, and the output responses x_i for all degrees-of-freedom i, except at the pth location

$$\begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{p-1} \\ x_{p+1} \\ \vdots \\ x_N \end{Bmatrix} = \left(\begin{Bmatrix} H_{1j} \\ H_{2j} \\ \vdots \\ H_{(p-1)j} \\ H_{(p+1)j} \\ \vdots \\ H_{Nj} \end{Bmatrix} - \frac{H_{pj}}{H_{pq}} \begin{Bmatrix} H_{1q} \\ H_{2q} \\ \vdots \\ H_{(p-1)q} \\ H_{(p+1)q} \\ \vdots \\ H_{Nq} \end{Bmatrix} \right) f_j \quad (5\text{-}1)$$

which can be expressed more succinctly in the form,

$$x_i = \left(H_{ij} - \frac{H_{iq}H_{pj}}{H_{pq}} \right) f_j \quad (5\text{-}2)$$

Equation (5-2) defines the complete set of receptance functions for the fictitious system whose poles are the zeros of the original system. The mode shape vectors of the fictitious system can be extracted from equation (5-2), which gives the ith column of the receptance matrix of the fictitious system. This result provides additional information, namely, the poles and mode shape vectors of the system in a different configuration than the physical test. It therefore constitutes an expansion of the knowledge-space for model updating or sensitivity analysis. The process of

LA-UR-01-5820, Unclassified

obtaining the additional data is called a pseudo-test. An interesting question now arises: Where does the new information come from?

The natural frequencies and mode shape vectors of the original system are extracted in the frequency range of the test but the original frequency responses also contain smaller contributions from the out-of-range modes. The out-of-range modes are therefore the source of the additional information, so that, in the fictitious system one may aim to excite modes that are not completely given by a linear combination of the modes extracted initially from the original system.

The idea of the pseudo-test was extended by Gérard Lallement to the case of simultaneous antiresonance in different receptance functions—typically, $h_{pq}(s)$ between degrees-of-freedom *(p; q)* and $h_{rs}(s)$ between degrees-of-freedom *(r; s)*—by applying constraints $x_p=0$ and $x_r=0$ at the pth and r^{th} degrees-of-freedom, respectively, by means of control forces f_q and f_s at the qth and s^{th} degrees-of-freedom, respectively (see Reference [C3] for details). The modified system with many simultaneous antiresonances is likely, in principle, to have modes richer in the additional information than the modes of a single antiresonance modified system. However, to obtain the receptance equations of the modified system it is necessary to invert a matrix of original-system receptance functions, the order of which is equal to the number of simultaneous antiresonance constraints. The matrix inversion is likely to be ill-conditioned if the out-of-range modes are at the noise level of the original receptance functions [C4].

It is important to notice that equations (5-1) and (5-2) constitute a predictive model with considerable advantages over other model types. Finite element modeling contains assumptions in the constitutive equations, approximations at joints and boundary conditions and discretization errors, to name only a few common sources of errors. Modal models are based on a truncated system of modes. But the receptance model represented by equations (5-1) and (5-2) is based on measurements without truncation, limited only by the level of noise on the measurements. This means that equations (5-1) and (5-2) potentially have many other applications in addition to model updating. John Mottershead and Gérard Lallement consider in Reference [C5] the modification of a structure to cause the creation of a node by the mutual cancellation of a pole and a zero. Gérard Lallement and Scott Cogan presented an overview paper at the NATO Advanced Study Institute in Portugal in 1998 [C6].

6. MODAL TESTING AND ANALYSIS

The purpose of this section is to illustrate how relevant the work of Gérard Lallement is to future industrial applications. To do so, the example of aerospace engineering is selected. Future space missions and their requirements in terms of structural dynamics are discussed. The discussion is restricted to the fields of modal testing and analysis.
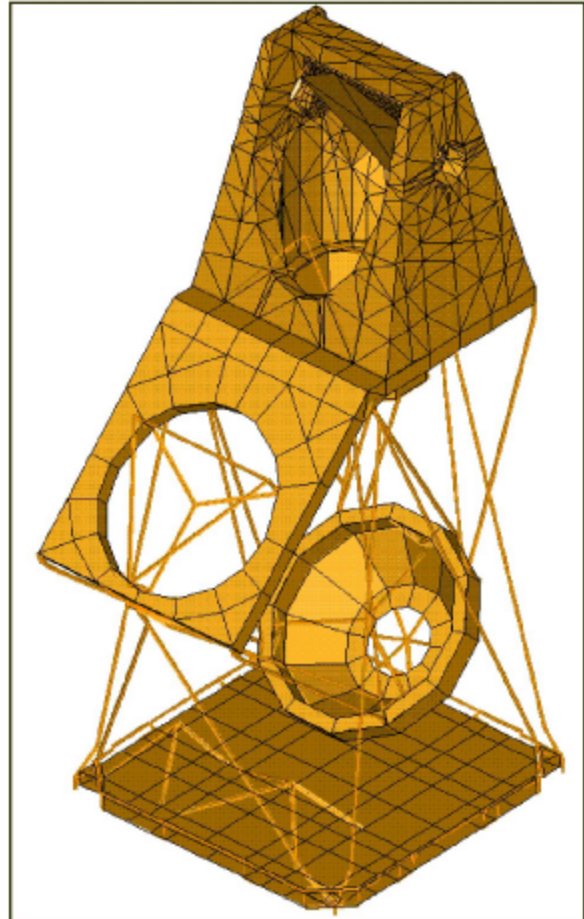
Figure 6. Modeling of the HRG telescope.
The figure shows the computational mesh of the high graphical resolution (HRG) telescope mounted on the SPOT satellite. Courtesy of CNES.

Gossamer structures are large, ultra-lightweight systems packaged into a small launch volume that promise reductions in mission cost. They will enable a wide variety of future missions requiring very large telescopes, antenna, large concentrators and sails or shades. Also included as part of Gossamer structures are large precision space structures. NASA has recognized the need for evolutionary and revolutionary technology development by soliciting and funding ideas for research and candidate flight experiments in the past few years.

11

6.1 Measuring and Predicting Modal Characteristics

The knowledge of the modal characteristics of a Gossamer structure is important to establish its ability to meet its mission requirements. Recent programs within NASA and other agencies have been aimed at providing experimental and analytical prediction of the modal characteristics of scale models of several Gossamer type structures. For example, flight experiments have been developed to predict or establish in-space modal characteristics.

The results indicate that the one-g, atmosphere, thermal simulation limitations on the ground severely limit the ability to accurately determine the in-space modal characteristics. At the same time, analytical predictions are suspect because the structural characteristics are significantly affected by the Earth' environment and other uncertainties that are difficult to establish. Improving the accuracy of numerical models and enabling the propagation of uncertainty through the simulations require that computational techniques such as finite element model updating, pseudo-testing and model condensation be integrated to the analysis tools. These are the same areas of research that Gérard Lallement has studied throughout his career.

6.2 Validation Experiments

The objectives of validation tests are not often clear amongst the researchers, engineers and program managers. The researcher and engineer's goal is to obtain the most accurate data of the test structure on the ground and obtain good correlation with the predictions. The program manager's goal is to validate that the hardware will meet the design objectives.

One of the most important objectives may be to "discover" unpredicted characteristics of the flight hardware that is significant to flight success. The value of this information is that it can later be used to modify and improve the design. Unfortunately, ground effects mask the small but significant factors that affect the modal parameters. Section 5 illustrates how the theory of zero and pole placement can be extended to develop numerical tools for pseudo-testing. Such virtual testing environment would enable the investigation of future testing scenarios. Because it is solely based on existing data sets and complemented with numerical simulations, pseudo-testing can be performed at a fraction of the cost and time required to instrument a prototype structure.

Nevertheless, many more challenges will need to be addressed before a sufficient predictive capability can be developed for analyzing engineering systems. For example, current state-of-the-art approaches of performing ground tests, developing a mathematical model that correlates with ground tests and then predicting on-orbit performance by subtracting the ground effects will not be adequate to validate the performance of Gossamer structures. Almost no new innovative ground test approaches are proposed.

6.3 Adaptive Structures

Another concept that has been proposed for many applications, among which future spacecraft missions, is the concept of adaptive structures. Although this is still work-in-progress to a large extent, an adaptive structure is generally designed to provide options to assure the system has the necessary modal parameters to successfully meet the mission requirements. The pre-flight validation objectives shift from precisely predicting and measuring the on-orbit modal characteristics by ground tests, to assuring that the design has the degree of adaptability in the hardware to encompass the uncertainties. Once the system is placed into orbit in its operational environment, the structure is adjusted to achieve its desired modal characteristics.

One particular type of adaptability consists of designing a controllable structure that is, by definition, able to change to make itself more controllable. For instance, undetected gaps in the joints that make the structure chaotic can be eliminated in space to allow it to respond linearly or as normal modes. Other examples are to eliminate identical eigenvalues, to change the eigenvectors, to eliminate modal localization, to eliminate parametric excitation and to adapt to other unexpected phenomena that may complicate the control of structures. Because of weight and packaging constraints, it is then advantageous to employ internal active elements as excitation and sensors. Such instrumentation and monitoring system can be used to obtain accurate modal characteristics of a free-free structure in space (on-orbit modal testing). Damping of the structure, eigenvalues and mode shape vectors can be reliably changed using the same active elements used for on-orbit modal testing.

Gérard Lallement has been among the first in the mechanical engineering community in Europe to recognize this priority by contributing to the development in 1999 of a research team for the study, manufacturing and integration of micro-electro-mechanical systems (MEMS) at the University of Besançon. The MEMS are micro-scale mechanical devices that can be designed to accomplish specific

tasks such as sensing and local actuation. Because of their extremely small size and power consumption, they offer distinctive advantages for integration in many engineering applications.

With the capability to measure and modify the structure's dynamic characteristics to the desired value in-space, the requirement for pre-flight measurement, analysis and validation of the dynamic characteristics substantially diminishes. An important benefit is the potential to validate the modal characteristics of Gossamer structures prior to flight.
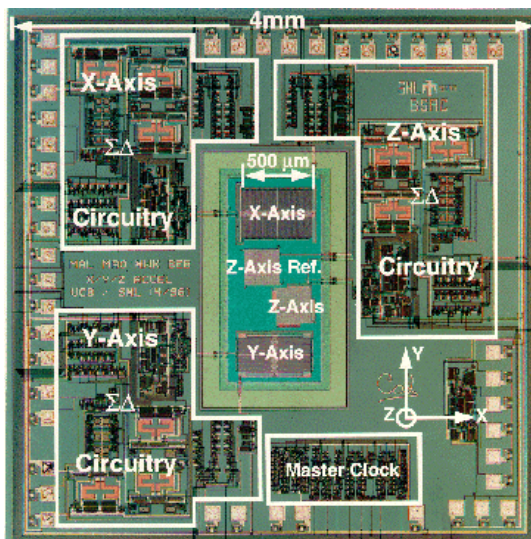
Figure 7. Example of MEMS device.
The figure illustrates a three-axis accelerometer with the integration of A/D converters, synchronization and all circuitry. Board dimensions are 4x4 mm². Courtesy of IMI Corporation, http://www.imi-mems.com.

7. CONCLUSION

This publication offers a tribute to Professor Gérard Lallement, from the University of Franche-Comté, France, for an outstanding 40 years (1961-2001) of research and development in the fields of experimental mechanics, mechanical engineering and structural dynamics. Throughout his fruitful career, Gérard Lallement's has authored more than 60 journal publications, presented more than 120 conference communications and offered numerous lectures in his areas of expertise.

Some of the areas of research with which Gérard Lallement has been most involved include modal analysis, structural system identification, the theory and practice of structural modification, component mode synthesis and finite element model updating. The technical aspects of Gérard Lallement's work are presented with a discussion of structural modification

and modeling error localization. The significance of his work is also illustrated by discussing the role of structural dynamics in industrial applications and future space missions.

The many of us who have had the privilege to cross paths with Gérard Lallement and got to know him both professionally and personally recognize, not just his outstanding technical expertise, but also his unbounded enthusiasm and profound humanity. He has always taken great pride in his work and has never failed to support and encourage his fellow colleagues in their endeavors. We know that his technical expertise and friendship are recognized and appreciated across the boundaries of generation, culture and nationality. Gérard Lallement has been and will remain a true source of inspiration to many of us. We all wish him a pleasant, well-deserved retirement but also know that, whether it is at the University of Besançon or elsewhere, he will be greatly missed.

REFERENCES

(A) General:

[A1] Lallement, G., **Rapport d'Activité 2000-20001 du Laboratoire de Mécanique Appliquée R. Chaléat**, *Activity Report*, University of Franche-Comté, France, Jun. 2001 (in French).

(B) Modeling Error Localization:

[B1] Bugeat, L.P., Fillod, R., Lallement, G., Piranda, J., "Adjustment of Conservative Non-gyroscopic Models From Measurements," *Shock and Vibration Bulletin*, Vol. 48, Part 3, Sep. 1978, pp. 78-81.

[B2] Lallement, G., "Localization Techniques," *Proceedings of the Workshop on Structural Safety Evaluation Based on System Identification Approaches*, Vieweg International Science Book Series, Vieweg and Sons, Braunschweig, Germany, 1988, pp. 212-233.

[B3] Zhang, Q., Lallement, G., "Dominant Error Localization in a Finite Element Modal Test of a Mechanical Structure," *Journal of Mechanical Systems and Signal Processing*, Vol. 1, No. 2, 1987, pp. 141-149.

[B4] Fillod, R., Lallement, G., Piranda, J., "Parametric Correction of Regular Non-dissipative Finite Element Models," *1st International Modal Analysis Seminar*, Leuven, Belgium, 1986.

[B5] Cogan, S., Lallement, G., "Matching Finite Element Models to Modal Data," *International Conference on Spacecraft Structures and Mechanical Testing*, Report ESA-SP-321, Noordwijk, The Netherlands, Apr. 1991.

[B6] Lallement, G., Andriambololona, H., "Localization of Errors in Computational Models Using Dynamic Test Data," *EURODYN-90 Conference*, Bochum, Germany, Jun. 1990.

[B7] Lallement, G., Cogan, S., Rade, D.A., "Procedures for Updating Dynamic Mathematical Models," *European Space Agency, ESTEC*, Contract Report 10133-92-NL-PP, Vol. 2, Part B, 1994.

(C) Fictitious Modifications and Pseudo-tests:

[C1] Zhang, Q., Lallement, G., "Selective Structural Modifications: Applications to the Problems of Eigen-solutions Sensitivity and Model Adjustment," *Journal of Mechanical Systems and Signal Processing*, Vol. 3, 1989, pp. 55-69.

[C2] Lallement, G., Cogan, S., "Reconciliation Between Measured and Calculated Dynamic Behaviors: Enlargement of the Knowledge Space," *10th International Modal Analysis Conference*, 1992, pp. 487-493.

[C3] Rade, D.A., Lallement, G., "A Strategy for the Enrichment of Experimental Data as Applied to an Inverse Eigen-sensitivity-based FE Model Updating Method," *Journal of Mechanical Systems and Signal Processing*, Vol. 12, 1998, pp. 293-307.

[C4] Cogan, S., Lallement, G., "Improvement of the Numerical Conditioning of a Structural Modification Procedure," *9th International Modal Analysis Conference*, Feb. 1991, pp. 279-303.

[C5] Mottershead, J.E., Lallement, G., "Vibration Nodes and the Cancellation of Poles and Zeros by Unit-rank Modifications to Structures," *Journal of Sound and Vibration*, Vol. 222, 1999, pp. 833-851.

[C6] Lallement, G., Cogan, S., "Parametric Identification Based on Pseudo-tests," *NATO Advanced Study Institute*, Sesimbra, Portugal, May 1998.

(D) Chronometry and the Measurement of Time:

[D1] Lallement, G., "Etude Théorique et Expérimentale de l'Echappement Magnétique," *Annales Françaises de Chronométrie*, No. 1, 1964, pp. 19-43 (in French).

[D2] Lallement, G., "Nouvel Oscillateur de Torsion," *Annales Françaises de Chronométrie*, No. 3, 1964, pp. 197-204 (in French).

[D3] Lallement, G., Leccanet, H., "Etudes Concernant les Suspensions de Diapasons et Certaines Réalisations Particulières," *Annales Françaises de Chronométrie*, No. 2, 1967, pp. 39-45 (in French).

[D4] Lallement, G., Oudet, C., "Etude de l'Entretien d'Oscillateurs Mécaniques par Transducteurs Piézoélectriques Fonctionnant en Accéléromètre," *Annales Françaises de Chronométrie*, No. 5, 1970, pp. 23-30 (in French).

(E) Inelasticity and Magneto-mechanical Coupling:

[E1] Lallement, G., Oytana, C., "Etude du Facteur de Surtension d'Oscillateurs Métalliques," *Annales Françaises de Chronométrie*, No. 2, 1967, pp. 55-62 (in French).

[E2] Lallement, G., Oytana, C., "Influence de la Magnétostriction sur les Effets Elastiques du Second Ordre," CRAS, Vol. 274, No. 16, 1972, pp. 1243-1246 (in French).

[E3] Lallement, G., "Etude Physique des Variations de Longueur Avec la Torsion des Métaux," Rheologica Acta, Vol. 13, No. 3, 1974, pp. 467-476 (in French).

[E4] Lallement, G., Oytana, C., "Contribution à l'Etude de l'Effet Poynting," *Journal de Mécanique Théorique et Appliquée*, Vol. 5, No. 2, 1981, pp. 1-15 (in French).

(F) Finite Element Model Updating:

[F1] Lallement, G., "Identification Methods: Adjustment of Mathematical Models," *Invited Contribution*, **Identification of Vibrating Structures**, H.G. Natke, Editor, Springer-Verlag, Germany, 1982, pp. 179-194.

[F2] Cogan, S., Lallement, G., Ayer, F., "Updating of Linear Elastic Models With Modal Selective Sensitivity," *Journal of Inverse Problems in Engineering*, Vol. 2, 1995, pp. 29-47.

[F3] Natke, H.G., Lallement, G., Cottin, N., "Properties of Various Residuals Within Updating of Mathematical Models," *Journal of Inverse Problems in Engineering*, Vol. 1, 1995, pp. 329-348.

(G) Sensitivity Analysis:

[G1] Lallement, G., Zhang, Q., "Selective Structural Modifications: Applications to the Problems of Eigen-solution Sensitivity and Model Adjustment," *Journal of Mechanical Systems and Signal Processing*, Vol. 3, No. 1, 1989, pp. 55-69.

[G2] Beliveau, J.G., Cogan, S., Lallement, G., Ayer, F., "Iterative Least-squares Calculation for Modal Eigenvector Sensitivity," *AIAA Journal*, Vol. 34, No. 2, Feb. 1996, pp. 385-391.

(H) Model Reduction:

[H1] Cogan, S., Lallement, G., Ayer, F. Ben-Haim, Y., "Model Order Reduction by Selective Sensitivity," *AIAA Journal*, Vol. 35, No. 3, Mar. 1997, pp. 557-562.

[H2] Lallement, G., Ramanitranja, A., "Enlargement of the Frequency Validity Domain of the Guyan Transformation," *Journal of Inverse Problems in Engineering*, Vol. 7, 1999, pp. 581-602.

(I) Optimal Sensor Placement:

[I1] Lallement, G., Ramanitranja, A., Cogan, S., "Optimal Sensor Deployment: Application to Model Updating Problems," *Journal of Vibration and Control*, Vol. 4, No. 1, 1998, pp. 29-46.

(J) Component Mode Synthesis:

[J1] Cogan, S., Lallement, G., "A Component Mode Decoupling Technique," *Journal of GAMI*, No. 427, Sep. 1988, pp. 15-17.

[J2] Bouazzouni, A., Lallement, G., Cogan, S., "Selecting a Ritz Basis for the Reanalysis of the Frequency Response Functions of Modified Structures," *Journal of Sound and Vibration*, Vol. 199, No. 2, 1997, pp. 309-322.

[J3] Lenoir, D., Cogan, S., Lallement, G., "Optimal Test Planning for Model Updating Using Craig-Bampton Reduced Models," *2nd COST-F3 International Conference on Structural System Identification*, Keynote Lecture, Kassel, Germany, Sep. 2001, pp. 69-79.

(K) Vibration Testing and Identification:

[K1] Zhang, Q., Lallement, G., "Comparison of Normal Eigen-modes Calculation Methods Based on Identified Complex Eigen-solutions," *AIAA Journal of Spacecraft and Rockets*, Vol. 24, No. 1, Jan.-Feb. 1987, pp. 69-73.

[K2] Zhang, Q., Lallement, G., "Relations Between Left and Right Hand Side Eigenvectors in Non-self Adjoint Structures; Applications to Axi-symmetrical Rotors," *Journal of Mechanical Systems and Signal Processing*, Vol. 2, No. 1, 1989, pp. 97-103.

[K3] Aitrimouch, H., Lallement, G., Kozanek, J., "Construction of the Unobserved Part of the Square Dynamic Flexibility Matrix," *Journal of Sound and Vibration*, Vol. 204, No. 1, 1997, pp. 73-84.

(L) Damping Identification:

[L1] Zhang, Q., Lallement, G., "Estimation of Generalized Damping Matrix From the Identified Complex Eigen-solutions," *Journal of Structural Mechanics*, Vol. 15, No. 4, 1987, pp. 141-149.

[L2] Foltete, E., Gladwell, G.M., Lallement, G., "On the Reconstruction of a Damped Vibrating System From Two Complex Spectra II; Experiment," *Journal of Sound and Vibration*, Vol. 219, No. 5, 1999.